Global Existence and Blow-up for a Degenerate Parabolic System Come from Logarithmic Function

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Abstract In this paper a class of initial-boundary value problem of degenerate parabolic system come from logarithmic function was considered. The local existence of solution was proved by approximation method. The condition of global existence of the solution was obtained by the upper solution of the characteristic function and the blow-up condition and the upper bound of the blow-up time were obtained by using the characteristic function.

Keywords degenerate parabolic system; local solution; global solution; blow-up
1. 解的局部存在性

对于退缩抛物型方程组初边值问题解的局部存在性与唯一性，讨论的方法有几种 ... + kφ) \right] \alpha (Δ ϕ + f(ϕ)) = 0

考虑如下问题

$$u_0 = A_s(u_s) = \left\{ \begin{array}{ll}
(1 + u_s) \left[ \ln(1 + u_s) \right]^{α} u_s \geq ε \\text{x} \in \Omega \\& \\Omega \times (0,T) \\
(1 + \frac{ε}{2}) \left[ \ln(1 + \frac{ε}{2}) \right]^{α} u_s < \frac{ε}{2}
\end{array} \right. \right.

$$

$$(2)$$

$$A_s(u_s) \subset B_s(v_s)$$

$$\left\{ \begin{array}{ll}
(1 + v_s) \left[ \ln(1 + v_s) \right]^{β} v_s \geq ε \\text{x} \in \Omega \\& \\Omega \times (0,T)
\end{array} \right. \right.

$$

$$(3)$$

$$A_s(u_s) \supset B_s(v_s)$$

首先利用逼近的方法给出解的局部存在性，其次讨论解整体存在的条件，最后讨论解在有限时刻发生爆破的条件。

2. 解的存在性

为了解的存在性，考虑如下问题

$$u_0 = A_s(u_s) = \left\{ \begin{array}{ll}
(1 + u_s) \left[ \ln(1 + u_s) \right]^{α} u_s \geq ε \\text{x} \in \Omega \\& \\Omega \times (0,T)
\end{array} \right. \right.

$$

$$(4)$$

$$A_s(u_s) \subset B_s(v_s)$$

$$\left\{ \begin{array}{ll}
(1 + v_s) \left[ \ln(1 + v_s) \right]^{β} v_s \geq ε \\text{x} \in \Omega \\& \\Omega \times (0,T)
\end{array} \right. \right.

$$

$$(5)$$

$$A_s(u_s) \supset B_s(v_s)$$

首先对于经典的方法还可以得到解的唯一性，当

$$u_0 = A_s(u_s) = \left\{ \begin{array}{ll}
(1 + u_s) \left[ \ln(1 + u_s) \right]^{α} u_s \geq ε \\text{x} \in \Omega \\& \\Omega \times (0,T)
\end{array} \right. \right.

$$

$$(6)$$

$$A_s(u_s) \subset B_s(v_s)$$

$$\left\{ \begin{array}{ll}
(1 + v_s) \left[ \ln(1 + v_s) \right]^{β} v_s \geq ε \\text{x} \in \Omega \\& \\Omega \times (0,T)
\end{array} \right. \right.

$$

$$(7)$$

$$A_s(u_s) \supset B_s(v_s)$$

由解的延展知
\((1 + k\phi) \ln (1 + k\phi) \geq \lambda, k\phi - f(\lambda \phi) \geq 0\) \\
(1 + k\phi) \ln (1 + k\phi) \geq (a_k \phi) \geq (1 + k\phi) \ln (1 + k\phi) \geq 0.

(3) \\
\begin{align*}
& \forall \phi \not\in \{1, \phi \} \ln (1 + \phi) \geq \phi(\lambda_1 k\phi - f(l \phi)) = \\
& \ln (1 + k\phi) \ln (1 + l \phi) \ln (a_k \phi) \geq \ln (1 + a_k \phi) \ln (a_k \phi) \geq 0.
\end{align*}

3

\[ u_0(x) = \phi(x) \int_{\Omega} (x) \geq \phi(x) \geq 0, \]

(4)

\[ \int u(x) = u(x) \phi(x) = \phi(x) \int_{\Omega} (x) \geq \phi(x) \geq 0. \]

(5)

\[ u(x) \phi(x) = \phi(x) \int_{\Omega} (x) \geq \phi(x) \geq 0. \]

(6)

\[ z_i = 1 + u \int (1 + u) \ln (1 + u) \geq \phi(x) \geq 0. \]

(7)

\[ z_i = 1 + u \int (1 + u) \ln (1 + u) \geq \phi(x) \geq 0. \]

\[ F'(t) = \int (\lambda \phi) \geq \int (\lambda \phi) \geq 0. \]

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