# On $k$-trees with Extremal Signless Laplacian Estrada Index and Estrada Index

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**Abstract** The signless Laplacian Estrada index :,YY(H) (resp. Estrada index YY(H)) of a graph H is defined as :,YY(H) ] $\sum_{\# ] 1}^{"}$ %$^{N_\#}$ (resp. YY(H) ] $\sum_{\# ] 1}^{"}$ %$^{\lambda_\#}$). Let $T^+_{"}$ be the set of the +-trees of order ". In this paper, by the methods on power series in mathematical analysis and spectral moment in algebra graph theory, the pseudo-order on the two indices was obtained. Further by contradiction, the extremal graphs among $T^+_{"}$ having the first, the second largest signless Laplacian Estrada index (resp. Estrada index) were characterized.

**Keywords** +-tree; signless Laplacian Estrada index; Estrada index
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## +-树的极值无符号的拉普拉斯 Estrada 指标和 Estrada 指标

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**摘 要** 图 H 的无符号的拉普拉斯 Estrada 指标 :,YY(H)(Estrada 指标 YY(H))定义为 :,YY(H) ] $\sum_{\# ] 1}^{"}$ %$^{N_\#}$(YY(H) ] $\sum_{\# ] 1}^{"}$ %$^{\lambda_\#}$).设 $T^+_{"}$ 为 " 阶 +-树的集合.利用数学分析中幂级数和代数图论中谱距的方法,建立了这两类指标的伪序,结合反证法,刻画了 $T^+_{"}$ 中具有第一、第二最大的无符号的拉普拉斯 Estrada 指标(Estrada 指标)的极值图.

**关键词** +-树;无符号拉普拉斯 Estrada 指标;Estrada 指标

All graphs considered in this paper are finite, undirected and simple. Let H=(W,Y) be a connected graph, let $U_H$(‘)={/|/‘∈Y},$U_H$[‘]=$U_H$(‘)∪{‘}. Denote $8_H$(‘)=|$U_H$(‘)| by the degree of the vertex ‘ of H. If B⊂W, let $U_H$(B)=$\cup_{‘\in B}$$U_H$(‘)\B,$U_H$[B] be the set of vertices within distance at most 1 from B, H−B be the subgraph of H obtained by deleting the vertices of B and the edges incident with them. If $Y_0$⊂Y(H), we denote by H−$Y_0$ the subgraph of H obtained by deleting the edges in $Y_0$. If $Y_1$ is the subset of the edge set of the complement of H,H+Y denotes the graph obtained from H by adding the edges in $Y_1$: If Y ={EF} and B={‘}, we write H−EF and H−‘ instead of H−{EF} and H−{‘}, respectively. The join $H_1 \vee H_2$ of two edge-disjoint graphs $H_1$ and $H_2$ is obtained by adding an edge from each vertex in $H_1$ to each vertex in $H_2$. For other undefined notations we refer to Bollobás[1].

The adjacency matrix &=&(H) of H is a matrix whose (#,l)-th entry is equal to 1 if vertices $‘_\#$ and $‘_l$ are adjacent, and 0 otherwise. Since & is a real symmetric matrix, its eigenvalues are real numbers. Let $\lambda_1 \geqslant \lambda_2 \geqslant \cdots \geqslant \lambda_{"}$ be the eigenvalues of adjacency matrix &. Let $=_+$(H) be the +th spectral moment of the

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graph H, i.e., $=_+(H)$ ] $\sum_{\#]1}^{"} \lambda_\#^+$ . The Estrada index, put forward by Estrada[2], is defined as YY(H) ] $\sum_{\#]1}^{"} \%^{\lambda_\#}$ ] $\sum_{+]0}^{\infty} \frac{=_+(H)}{+!}$.

Let G = G(H) = 8#4$(8('$_1$),…,8('$_"$)) be a diagonal matrix with degrees of the corresponding vertices of H on the main diagonal and zero elsewhere, where 8('$_\#$) is the degree of '$_\#$. The matrix @ = G(H) + &(H) is called the signless Laplacian of H. Since @ is real symmetric and positive semi-definite matrix, its eigenvalues are real numbers. Let $N_1 \geqslant N_2 \geqslant \cdots \geqslant N_" \geqslant 0$ are the signless Laplacian eigenvalues of H. The multiplicity of 0 as an eigenvalue of @ is equal to the number of bipartite connected components of H. The set of all eigenvalues of @ is the signless Laplacian spectrum of H.

A semi-edge walk (of length +) in a graph H is an alternating sequence '$_1$, %$_1$, '$_2$, %$_2$, …, '$_+$, %$_+$, '$_{++1}$ of vertices '$_1$, '$_2$, …, '$_{++1}$ and edges %$_1$, %$_2$, …, %$_+$ such that for any # = 1, 2, …, + the vertices '$_\#$ and '$_{\#+1}$ are end-vertices (not necessarily distinct) of the edge %$_\#$. From Ref [3], we know that the (#, L)-entry of the matrix $@^+$ is equal to the number of semi-edge walks of length + starting at vertex '$_\#$ and terminating at vertex '$_L$. Let $S_+$ ] $\sum_{\#]1}^{"} N_\#^+$ ] S0($@^+$) be the +-th spectral moment for @, that is, $S_+$ is equal to the number of closed semi-edge walks of length +. Then the signless Laplacian Estrada index :,YY(H), which is introduced by Ayyaswamy S K, et al in Ref [4], of a graph H can be written as :,YY(H) ] $\sum_{\#]1}^{"} \%^{N_\#}$ ] $\sum_{+]0}^{\infty} \frac{S_+}{+!}$.

As an extension of trees, the +-tree is defined as either a complete graph on + vertices or a graph obtained from a smaller +-tree by adjoining a new vertex together with + edges connecting it to a + + 1-clique. Let $T_"^+$ be the set of all "-vertex +-trees.

Since Harary and Palmer[5] first introduced 2-trees in 1968, Beineke and Pippert[6] gave the definition of a +-tree in 1969, the study of properties on +-trees arouse many researchers interest. For examples, Estes J and Wei B[7] obtained the sharp bounds of the Zagreb indices of +-trees. In Ref [8], Wang S and Wei B characterized the extremal graphs and determined the exact bounds of multiplicative Zagreb indices of +-trees, which attain the lower and upper bounds. In Ref [9], Wang X X, et al attained the upper bounds on the spectral radius of +-trees. Huang F and Wang S[10] characterized the graphs among $T_"^+$ having the first (resp. the second) largest Estrada index, and so on. In this note, we will give a unified approach, which is simpler than the one used in Ref [10], to characterize the graphs among $T_"^+$ having the first, the second largest signless Laplacian Estrada index (resp. Estrada index).

## 1 Preliminaries

In this section, we give some definitions and structure properties of +-trees which will be used in the proof of our main results.

Let $S_"^+$ be a +-tree. If ' is a vertex of $S_"^+$ with degree + whose neighbours form a +-clique, then ' is called a +-simplicial vertex. We use :$_1$($S_"^+$) for the set of all simplicial vertices of $S_"^+$, when " ⩾ + + 2. Set :$_1$($Q_+$) = ∅ and :$_1$($Q_{++1}$) = {'}, where ' is any vertex of $Q_{++1}$.

**Lemma 1**[11]

(i) :$_1$($S_"^+$) ≠ ∅ for " ⩾ ++1.

(ii) :$_1$($S_"^+$) is an independent set when " ⩾ ++1.

(iii) $S_"^+$ − :$_1$($S_"^+$) is a +-tree and every +-simplicial vertex (if any) of $S_"^+$ − :$_1$($S_"^+$) is adjacent in $S_"^+$ to at least one vertex of :$_1$($S_"^+$).

Let $Q_+$ be a +-clique and : be an independent set of " − + vertices. &(+;")-star, denoted by :$_{+,"-+}$, is defined as :$_{+,"-+}$ = $Q_+$ ∨ : (as shown in Fig. 1). Let :T$_{+,"-+}$ = :$_{+,"-+}$ − /$_1$'$_1$ + /$_1$/$_2$.

**Lemma 2**[10] Let $S_"^+$ be a +-tree of order ". If |:$_1$($S_"^+$)| = " − +, then it is a (+;")-star.

Let H and 9 be two graphs with /$_1$, '$_1$ ∈ W(H) and /$_2$, '$_2$ ∈ W(9). If $=_+$(H; /$_1$, '$_1$) ⩽ $=_+$(9; /$_2$, '$_2$) for all positive integers +, then we write (H; /$_1$, '$_1$) ⩽ =(9; /$_2$, '$_2$). If (H; /$_1$, '$_1$) ⩽ =(9; /$_2$, '$_2$) and there is at least one positive integer $+_0$ such that $=_{+_0}$(H; /$_1$, '$_1$) < $=_{+_0}$(9; /$_2$, '$_2$); then we write (H; /$_1$, '$_1$) < =(9; /$_2$, '$_2$). Similarly, if $S_+$(H; /$_1$, '$_1$) ⩽ $S_+$(9; /$_2$,

$v_2$) for all positive integers +, then we write (H;/$_1$,'$_1$) ≤S(9;/$_2$,'$_2$). If (H;/$_1$,'$_1$) ≤S(9;/$_2$,'$_2$) and there is at least one positive integer $+_0$ such that $S_{+_0}$(H;/$_1$,'$_1$) <$S_{+_0}$(9;/$_2$,'$_2$); then we write (H;/$_1$,'$_1$) <S(9;/$_2$,'$_2$). If /$_\#$ = '$_\#$, we write (H;/$_\#$,'$_\#$) as (H;/$_\#$) for # = 1,2.

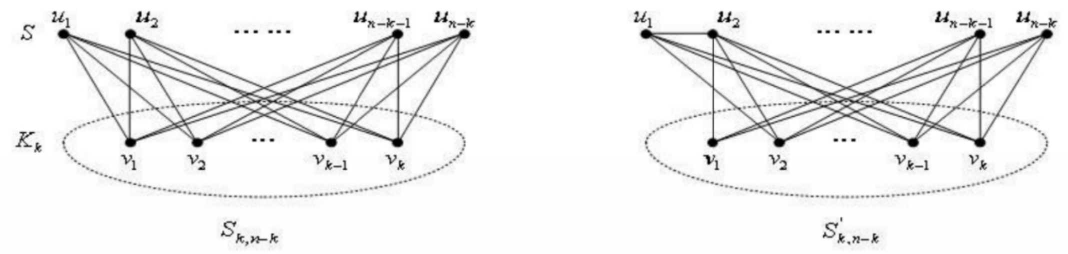


Fig.1 The +-trees :$_{+,"-+}$ and :⊤$_{+,"-+}$

**图 1 +-树 :$_{+,"-+}$和 :⊤$_{+,"-+}$**

**Lemma** 3 Let /,' ∈ W(H) and $U_H$(') ⊆ $U_H$ [/]. Then

(i) (H;') ≤ =(H;/), and (H;;,') ≤ =(H;;,/) for each ; ∈ W(H). Moreover, if $8_H$(') <$8_H$(/) then (H;') < =(H;/).

(ii) (H;') ≤S(H;/), and (H;;,') ≤S(H;;,/) for each ; ∈ W(H)[12]. Moreover, if $8_H$(') < $8_H$(/), then (H;') <S(H;/).

**Proof** We only need to prove (i).

Firstly, we prove (H;') ≤ =(H;/).

Let B be a walk in $B_+$(H;'). If / is not in B, note that $U_H$(') ⊆ $U_H$ [/], let >(B) = B⊤, where B⊤ is the walk that is obtained by replacing the vertex ' by /, obviously, B⊤ ∈ $B_+$(H;/). If / is in B, we can also look B as a walk which is starting and ending at vertex /, let >(B) = B.

Obviously, > is an injection from $B_+$(H;') to $B_+$(H;/). Hence (H;') ≤ =(H;/). If $8_H$(') < $8_H$(/), > is not a surjection, then (H;') <S(H;/).

Secondly, we prove (H;;,') ≤ =(H;;,/) for each ; ∈ W(H). If $\widetilde{B}$ be a walk in $B_+$(H;;,'), let $\widetilde{>}$($\widetilde{B}$) = $\widetilde{B}$⊤, where $\widetilde{B}$⊤ is the walk which is obtained by replacing the vertex ' by /. $\widetilde{>}$ is also an injection $B_+$(H;;,') to $B_+$(H;;,/), we have (H;;,') ≤ =(H;;,/) for each ; ∈ W(H).

**Lemma** 4 Let H be a graph and ',/,;$_1$,;$_2$,⋯,;$_0$ ∈ W(H). Suppose that Y. = {%$_1$ = ';$_1$,⋯,%$_0$ = ';$_0$} a Y$_/$ = {%⊤$_1$ = /;$_1$,⋯,%⊤$_0$ = /;$_0$}, where %$_\#$,%⊤$_\#$ ∉ Y(H), for # = 1,2,⋯,0, Let H$_/$ = H+Y$_/$ and H. = H+Y..

(i) If (H;') <S(H;/), and (H;;$_\#$,') ≤ =(H;;$_\#$,/) for each # = 1,2,⋯,0, then YY(H.) <YY(H$_/$)[13].

(ii) If (H;') <S(H;/), and (H;;$_\#$,') ≤S(H;;$_\#$,/) for each # = 1,2,⋯,0, then :,YY(H.) <:,YY(H$_/$)[14].

Lemma 4 is an excellent tool to deal with the extremal problems on Estrada index and signless Laplacian Estrada index, but it has many conditions which have to be provided when we want to use it. The lemma 3 enables us to discover a special case that provides such conditions.

## 2 Main results

In this section, we will give a unified method to characterize the +-trees with the largest and second largest Estrada index and signless Laplacian Estrada index, respectively, which is simpler than the method provided in Ref [10].

**Lemma** 5 Let H be a graph with '/,/; ∈ Y(H) and '; ∉ Y(H). Let H⊤ = H−'/+';. If $U_H$(') ⊆ $U_H$ [/], then YY(H) <YY(H⊤) and :,YY(H) <:,YY(H⊤).

**Proof** Let 9 = H−'/. Then H = 9+'/; H⊤ = 9+'; and $U_9$(') ⊆ $U_9$ [/]. Note that ; ∉ $U_H$ ['], we have $8_9$(') <$8_9$(/). By Lemma 3, we have (9;;,') ≤ I(9;;,/) for each ; ∈ W(9) and (9;') < I(9;/), where I ∈ {=,S}. Further by Lemma 4, we have YY(H) < YY(H⊤) and :,YY(H) <:,YY(H⊤).

Repeated by Lemma 5, we can obtain the following result.

**Lemma** 6 Let H be a graph and 7 = {E$_1$,E$_2$,⋯,E$_R$} be an independent set of equivalent vertices such that E$_\#$/,/; ∈ Y(H) and E$_\#$; ∉ Y(H) for 1 ≤ # ≤ R. Let H⊤ = H−{E$_\#$/,1 ≤ # ≤ R}+{E$_\#$;,1 ≤ # ≤ R}. If $U_H$(') ⊆ $U_H$ [/], then YY(H) <YY(H⊤) and :,YY(H) <:,YY(H⊤).

**Lemma** 7 For a graph H ∈ $T^+_{"}$, if there exists a vertex $‘ \in :_1(H-:_1(H))$, then there is a graph $HT \in \boldsymbol{T}^{+}_{"}$ such that $|:_1(HT)| = |:_1(H)|+1$, $YY(H) < YY(HT)$ and $:,YY(H)<:,YY(HT)$.

**Proof** Since there exists a vertex $‘ \in :_1(H-:_1(H))$, let $U_{H-:_1(H)}(‘) = \{‘_1, ‘_2, \cdots, ‘_+\}$, $!_H(‘) = U_H(‘) - U_{H-:_1(H)}(‘)$. Then the vertices in $U_{H-:_1(H)}(‘)$ induce a complete graph $Q_+$; and the vertices in $!_H(‘)$ which are all simplicial verices, induce an empty graph.

For a vertex $E \in !_H(‘)$, it is adjacent to all but one vertex in $U_{H-:_1(H)}(‘)$. Let $!_\#$ be the set of vertices in $!_H(‘)$ whose neighbour set is $P_H(‘) = |\{1 \leqslant \# \leqslant +, !_\# \neq \varnothing\}|$, where $1 \leqslant \# \leqslant +$. For a vertex $‘ \in :_1(H-:_1(H))$, let $‘ \in :_1(H-:_1(H))$ and $K(H) = \min|\{P_H(‘), ‘ \in :_1(H-:_1(H))\}|$. Without loss of generality, let $K_H(/) = K(H)$, $U_{H-:_1(H)}(‘) = \{‘_1, ‘_2, \cdots, ‘_+\}$ and $!_H(/) = !_1 \cup \cdots \cup !_{K(H)}$ (as shown in Fig. 2). Obviously, we have $U_H[!_\#] \subseteq U_H[/]$ for $1 \leqslant \# \leqslant +$. Let $HT = H - \{/E : E \in !_1\} + \{‘_1 E : E \in !_1\}$.

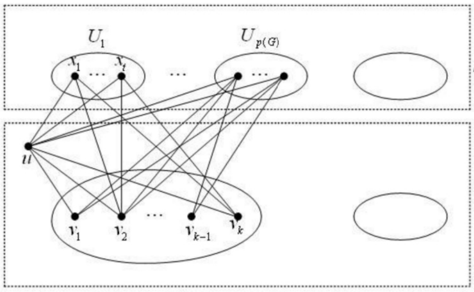


Fig.2　The structure of $U_{H-:_1(H)}(‘)$ and $!_H(/)$

图 2　$U_{H-:_1(H)}(‘)$和 $!_H(/)$的结构

Note that $‘_1E \notin Y(H)$. By Lemma 6, we have $YY(H)<YY(HT)$ and $:,YY(H) < :,YY(HT)$. Obviously, $:_1(HT) = :_1(H) \cup \{/\}$, and $|:_1(HT)| = |:_1(H)|+1$. This completes the proof.

Repeatedly by Lemma 7, we can see that the +-tree $S^{+}_{"}$ which has the largest Estrada index and signless Laplacian Estrada index satisfies $:_1(S^{+}_{"} - :_1(S^{+}_{"})) = \varnothing$. Further by lemmas 1 and 2, we have $S^{+}_{"} \cong :_{+,"-+}$. Then we have the following theorem.

**Theorem** 1　For any $S^{+}_{"} \in \boldsymbol{T}^{+}_{"}$, $YY(S^{+}_{"}) < YY(:_{+,"-+})$ and $:,YY(S^{+}_{"}) < :,YY(:_{+,"-+})$. The equality holds if and only if $S^{+}_{"} \cong :_{+,"-+}$.

In the following, we consider the graphs with the second largest Estrada index and signless Laplacian Estrada index in $\boldsymbol{T}^{+}_{"}$.

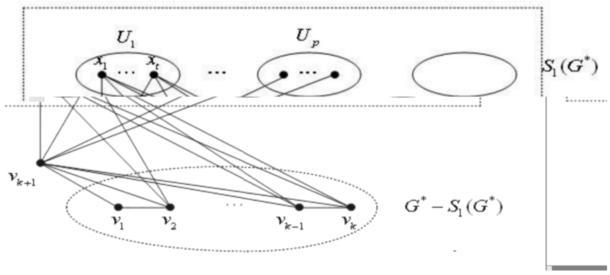


Fig.3　The structure of the graph $H^*$ in the proof in Theorem 9

图 3　定理 9 证明中图 $H^*$ 的结构

**Theorem** 2　For any $S^{+}_{"} \in \boldsymbol{T}^{+}_{"} \setminus :_{+,"-+}$, $YY(S^{+}_{"}) \leqslant YY(:T_{+,"-+})$ and $:,YY(S^{+}_{"}) \leqslant :,YY(:T_{+,"-+})$. The equality holds if and only if $S^{+}_{"} \cong :T_{+,"-+}$.

**Proof** Let $H^*$ be the graph in $\boldsymbol{T}^{+}_{"} \setminus :_{+,"-+}$ whose Estrada index and signless Laplacian Estrada index is as larger as possible. By Lemma 7, we have $|:_1(H^* - :_1(H^*))| = 1$, that is, $H^* - :_1(H^*) \cong Q_{++1}$.

Let $W(H^*) \setminus :_1(H^*) = \{‘_1, ‘_2, \cdots, ‘_{++1}\}$ and $|:_1(H^* - :_1(H^*))| = \{‘_{++1}\}$. Further, let $!_H(‘_{++1}) = U_H(‘_{++1}) - \{‘_1, ‘_2, \cdots, ‘_+\}$; $!_\#$ be the set of vertices in $!_H(‘_{++1})$ whose neighbour set is $\{‘_1, \cdots, ‘_{\#-1}, ‘_{\#+1}, \cdots, ‘_+, ‘_{++1}\}$ for $1 \leqslant \# \leqslant +$, and $K = |\{1 \leqslant \# \leqslant +, !_\# \neq \varnothing\}|$ (as shown in Fig.3).

**Case** 1　$K \geqslant 2$, let $H_1 = H^* - \{‘_{+-1}E : E \in !_1\} + \{‘_1 E : E \in !_1\}$. Obviously, $H_1 \in \boldsymbol{T}^{+}_{"} \setminus :_{+,"-+}$. By Lemma 6, we have $YY(H_1) > YY(H^*)$ and $:,YY(H_1) > :,YY(H^*)$, a contradiction.

**Case** 2　$K = 1$ and $:_1(H^*) - !_H(‘_{++1}) = \varnothing$, then $H^* \cong :_{+,"-+}$ it is contradict to $H^* \in \boldsymbol{T}^{+}_{"} \setminus :_{+,"-+}$.

**Case** 3　$K = 1$ and $:_1(H^*) - !_H(‘_{++1}) = \varnothing$.

If $|!_1| \geqslant 2$, let $!_1 = \{E_1, \cdots, E_R\}$ and $H_2 = H^* - ‘_{++1}E_1 + ‘_1 E_1$. Obviously, $H_2 \in \boldsymbol{T}^{+}_{"} \setminus :_{+,"-+}$. By Lemma 5, we have $YY(H_1) > YY(H^*)$ and $:,YY(H_2) > :,YY(H^*)$, a contradiction.

If $|!_1| = 1$, we have $H^* \cong :T_{+,"-+}$.

**Case** 4　$K = 0$, then $H^* \cong :_{+,"-+}$, it is contradict to $H^* \in \boldsymbol{T}^{+}_{"} \setminus :_{+,"-+}$.

This completes the proof.

For any $S^{+}_{"} \in \boldsymbol{T}^{+}_{"}$, if $+ = 1$, then $S^{+}_{"}$ is the ordinary tree. Let $\boldsymbol{T}_{"}$ be the class of trees of order ". Let $:_{"}$ and $:T_{"}$ be the graphs as shown in Fig.4. Then by theorems 1 and 2, we have the following results.

**Corollary**　Let $S \in \boldsymbol{T}_{"} \setminus \{:_{"}, :T_{"}\}$. Then

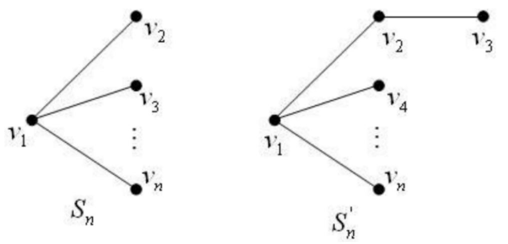


Fig.4 The graphs :ₙ and :Tₙ

图4 图:ₙ 和 :Tₙ

(i) YY(S)<YY(:Tₙ)<YY(:ₙ)[5].

(ii) :,YY(S)<:,YY(:Tₙ)<:,YY(:ₙ).

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